Collisions - II

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Collisions

- Total distance and time
- 2-D collisions
- Additional cases
- Multiple collisions

Repeated collisions with ground

Consider a body dropped from an initial height h_1. The body undergoes repeated inelastic collisions with the ground and finally comes to rest. Let e be the coefficient of restitution.

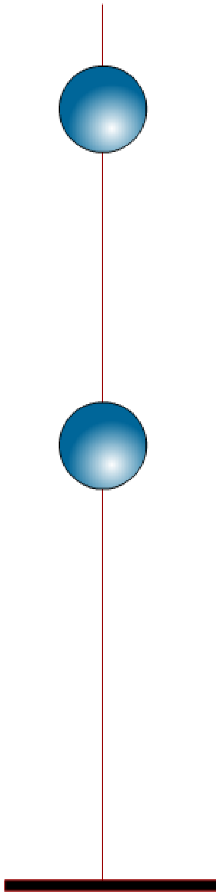
Coefficient of restitution is related to the initial and final heights as

$$\boxed{e = \sqrt{\frac{h_2}{h_1}}}$$

Height to which the body rises , after each collision is

$$h_{\mathrm{f}} = e^2 h_{\mathrm{i}}$$

After each collision the body reaches to a lesser height. This process continues and finally the body comes to rest in the ground.



Click here for simulation

Repeated collisions with ground

Total distance covered by the body

Height to which the body rises to , after each collision is

$$h_2 = e^2 h_1$$

$$h_3 = e^2 h_2$$

$$\Rightarrow h_3 = e^4 h_1$$

$$h_4 = e^2 h_3$$

$$\Rightarrow h_4 = e^6 h_1$$

After n collisions the height attained is

$$h_n = e^{2n} h_1$$

Total distance covered by the body is given by

$$S = h_1 + 2e^2 h_1 + 2e^4 h_1 + 2e^6 h_1 + 2e^{2n} h_1 \ldots$$

$$S = h_1 \left(1 + 2e^2 + 2e^4 + 2e^6 \ldots \right)$$

$$S = h_1 \left(1 + 2e^2 \left(1 + e^2 + e^4 + e^6 \ldots \right) \right)$$

Using the relation

$$1 + x + x^2 + x^3 \ldots = \left(\frac{1}{1 - x} \right)$$

where $x = e^2$ we get

$$\boxed{S = h_1 \left(1 + \frac{2e^2}{1 - e^2} \right)}$$

Repeated collisions with ground

Time taken by body to come to rest

TOD from initial height is

$$TOD\ (t) = \sqrt{\frac{2h_1}{g}}$$

After 1st collision

$$t_2 = 2\sqrt{\frac{2h_2}{g}} \quad \Rightarrow \quad t_2 = e2\sqrt{\frac{2h_1}{g}}$$

After 2nd collision

$$t_3 = 2\sqrt{\frac{2h_3}{g}} \quad \Rightarrow \quad t_3 = e^2 2\sqrt{\frac{2h_1}{g}}$$

After nth collision

$$t_n = 2\sqrt{\frac{2h_n}{g}} \quad \Rightarrow \quad t_3 = e^{2n} 2\sqrt{\frac{2h_1}{g}}$$

Total time of flight is given by

$$t_{\text{total}} = t + e2t + e^2 2t + e^3 2t\ldots$$

$$t_{\text{total}} = t\left(1 + e2 + e^2 2 + e^3 2\ldots\right)$$

$$t_{\text{total}} = t\left(1 + e2\left(1 + e + e^2\ldots\right)\right)$$

Using the relation

$$1 + x + x^2 + x^3\ldots = \left(\frac{1}{1-x}\right)$$

where $x = e$ we get

$$t_{\text{total}} = t\left(1 + \frac{2e}{1-e}\right)$$

$$\boxed{t_{\text{total}} = t\left(\frac{1+e}{1-e}\right)}$$

Repeated collisions with ground

Average velocity

Displacement of the body is

$$S = -h_1$$

Time taken by body to come to rest is

$$t_{\text{total}} = \sqrt{\frac{2h_1}{g}}\left(\frac{1+e}{1-e}\right)$$

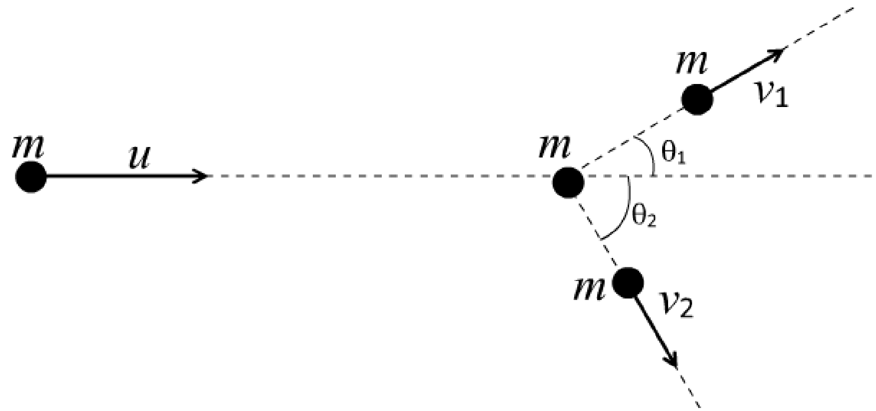
Using relations (i) and (ii) we get

$$v_{\text{avg}} = \frac{-h_1}{\sqrt{\frac{2h_1}{g}}\left(\frac{1+e}{1-e}\right)}$$

$$v_{\text{avg}} = -\sqrt{\frac{gh_1}{2}}\left(\frac{1-e}{1+e}\right)$$

2-D collisions

Let mass of each body be m and the initial velocity of the first body be u along the x-axis. Let the final velocities be v_1 and v_2 making angles θ_1 and θ_2 respectively w.r.t. the x-axis.



Using law of conservation linear momentum we get

$$m\bar{u} = m\bar{v}_1 + m\bar{v}_2$$

$$\bar{u} = \bar{v}_1 + \bar{v}_2$$

$$u^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2) \quad \text{--- (i)}$$

From conservation of kinetic energy we get

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$u^2 = v_1^2 + v_2^2$$

Using u^2 from equation (i)

$$v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2) = v_1^2 + v_2^2$$

$$\Rightarrow \quad 2v_1v_2\cos(\theta_1 + \theta_2) = 0$$

$$\Rightarrow \quad \cos(\theta_1 + \theta_2) = 0$$

$$\Rightarrow \quad \theta_1 + \theta_2 = 90^\circ$$